Special Relativity

without « invariance of c » postulate

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The usual derivation of Lorentz transformations, ie of special relativity, is based on the two following postulates
* **Postulate of Relativity** : the laws of the universe are the same regardless of inertial frame of reference
* **Invariance of c** : The speed of light in a vacuum is a universal constant (*c*) which is independent of the motion of the light source.

However, we will see this in this document that it is possible to derive Lorentz transformation without that second postulate. We will replace it (see Comment on page 2) by a more geometrical postulate, the cosmological principle, which says that :
***Cosmological Principle** : Space is homogenous and isotropic. Time is homogeneous.

ϵ is the distance between A and O' according to R
ϵ' is the distance between A and 0 according to R'.

We immediately get :

$$x=\epsilon+vt \quad (1)$$
$$x'=\epsilon'+v't' \quad (2)$$

Homogeneity of space requires that the relationship between the distance of the event from O′, measured in the two frames, be linear (see Proof at the end of the document) :

$$x'=\gamma_v \epsilon$$
$$x=\gamma'_{v'} \epsilon'$$

We of course have $\gamma_v , \gamma'_{v'} \geq 0$ as we are measuring distance, and are function of v,v'

Using this in (1) and (2), we get :

$$x'=\gamma_v (x-vt) \quad (3)$$
$$t'=\frac{-\gamma_v v}{v'}\left(t-\frac{\gamma_v \gamma'_{v'}-1}{\gamma_v \gamma'_{v'} v}x\right) \quad (4)$$

Isotropy of space and cosmological principle require that $v'=-v$
Relativity principle requires that $\gamma_v=\gamma'_{v'}$

So we can rewrite (3), (4)

$$x'=\gamma_v (x-vt) \quad (5)$$
$$t'=\gamma_v (t-K_1 x) \quad (6)$$

taking $K_1=\dfrac{\gamma_v^2-1}{\gamma_v^2 v}$

Now let's consider a third reference frame R" moving relatively to R' with a velocity u.

Making some algebra (as we did for R' and R but this time for R" and R and then putting R" in function of R) we find :

$$x''=\gamma_v \gamma_u [(1+K_1 u)x-(u+v)t]$$
$$t''=\gamma_v \gamma_u [-(K_1+K_2)x+(1+K_2 v)t]$$

The cosmological principle requires that $1+K_1 u=1+K_2 v$ which means that :

$$\frac{v}{K_1}=\frac{u}{K_2}=a$$

where "a" is aconstant that doesn't depend on u,v.
Putting this result in (5), (6), we get :

$$x'=\gamma_v(x-vt)$$

$$t'=\gamma_v(t-\frac{vx}{a})$$

But the relativity principle tells us that we must also have

$$x=\gamma_v(x'+vt')$$

$$t=\gamma_v(t'+\frac{vx'}{a})$$

Using those two sets of formula, we finally get :

$$\boxed{\gamma_v=\frac{1}{\sqrt{1-\frac{v^2}{a}}}}$$

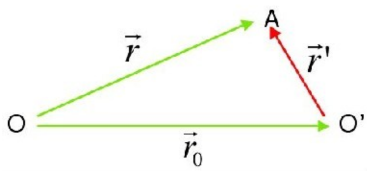
So we can write the well known Lorentz transformations, by putting $a=c^2$:

$$\boxed{x'=\frac{(x-vt)}{\sqrt{1-\frac{v^2}{c^2}}} \quad ; \quad t'=\frac{(t-\frac{vx}{c^2})}{\sqrt{1-\frac{v^2}{c^2}}}}$$

Comment
Even though this demonstration seems to be more powerful than the one based on c-invariance, as it is not supposing anything about the speed of light or any other physical quantity, there is a major criticism we can make on the second postulate.
The cosmological is a geometrical assumption, so it is purely mathematical. In other words, it doesn't have any real meaning in physics, unless it is associated with a measure instrument for which we assume it is verified. This means that if we want isotropy to be real and obervable, then we need to use an instrument that is isotropic and that gives the same result for any observer ! Which means that to verify the cosmological principle, we need to have something that has a constant speed in any inertial reference frame.

What we can see here is that cosmological principle and c-invariance are in fact two ways to see the same phenomenon : without any perfect instrument to make measurements (ie masless particle), saying that "space is isotropic, and my instrument changes" is the same to saying "my instrument works perfectly, and it showed me space is anisotropic".
Without any perfect measure instrument, space isotropy/homogeneity is equivalent to having an indeformable instrument.

Proof that cosmological principle leads to linearity



Let's suppose that we have $v_{\vec{A/O}}=f(\vec{r})$ we f is any kind of function. We then have :

$$v_{\vec{A/O'}}=f(\vec{r}')=f(\vec{r}-\vec{r}_0)$$

$$v_{\vec{A/O'}}=v_{\vec{A/O}}-v_{\vec{O'/O}}=f(\vec{r})-f(\vec{r}_0)$$

So we get that $f(\vec{r}-\vec{r}_0)=f(\vec{r})-f(\vec{r}_0)$ which means that $f(\vec{r})$ has to be a linear function.