On the clumpiness of dark matter, the formation of caustic rings and their possible detection : evaluation of neutralino annihilation signal

(August 10, 2006)

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Introduction

I. Introducing the lab

The 5 week traineeship was done from the 6th of July to the 10th of August 2006 at the Laboratoire d'Annecy-Le-Vieux de Physique THéorique (LAPTH), a mixed unit of research (UMR 5108) CNRS/Université de Savoie which is part of CNRS "Sciences Physiques et Mathématiques" department.
With 25 permanent members (15 CNRS, 6 teachers-researchers and 4 ITA/IATOS), 4 PhD students, 2 post-docs and many French and international guests, the LAPTH main fields of research are :

- Elementary particle Physics
- Cosmology / Particle Astrophysics
- Mathematical Physics, mostly field theories and symmetries.

My traineeship was done under the supervision of Richard Taillet, teacher-researcher in Astrophysics.

II. Introducing the subject

According to the latest results of the WMAP satellite **[1]**, combined with other cosmology data, it is now believed that only 4% of the Universe is made of ordinary (baryonic) matter ; another 22% is an as-yet unidentified dark matter, and 74% is a mysterious dark energy.

One of the major evidence for *dark matter* is the rotation curve of galaxies **[2]**, which is not as it should be according to Newtonian dynamics ; if we write Newton's equations governing the rotation of a galaxy, we see that the rotation curve we obtain (how speed evolves with radius) doesn't agree with the observation ; outer stars move faster than predicted. This problem can be solved if we suppose that there is matter we can't see (dark matter) that plays a key role in galactic dynamics. Other theories were proposed to solve the problem without supposing the existence of dark matter, like a modification of Newton's law, but they face serious problems to explain certain observations. The major candidates for dark matter are the neutralino and the axion ; the neutralino is a hypothetical particle predicted by supersymmetric theories, the axion is a hypothetical exotic subatomic particle postulated by Percei-Quinn theory to solve the strong CP problem in Quantum ChromoDynamics.

At the LAPTH, several scientists work on the possible ways to detect dark matter, directly or indirectly : dark matter particles may collide and annihilate, and we should be able to detect the product of these annihilations (an annihilation signal) which would give us information on what dark matter is made of. The intensity of the annihilation signal depends of dark matter properties : its microscopic properties (the way dark matter particles collide, annihilate,...) and its macroscopic properties (especially its spatial distribution). Does it form clumps or is it distributed homogeneously ? If it forms clumps, how could we be able to detect them ? How is it distributed on small-scales ?

To answer those questions, scientists simulated a collapsing dark matter halo**[3]** and found out that in most cases dark matter actually form clumps, which means that dark matter may not be distributed homogeneously. Moreover, recent works have shown that *under certain conditions*, dark matter also forms what is now called caustic rings **[4][5]**, which means that dark matter should be highly concentrated in some regions (we get a winding up in phase-space which indicates a sharply-peaked density)

During this 5 week traineeship, I have worked on the formation of caustic rings in a simplified framework. I have searched under what conditions caustic rings can form, evaluating how dense they should become, and how intense the neutralino annihilation signal should be. As in those highly-concentrated areas dark matter particles (neutralinos) are more likely to collide, we should be able to detect a neutralino annihilation peak in different predictable locations**[6]**. If dark matter forms clumps and can also form caustic rings, then its detection could become much easier than a homogeneously distributed halo of dark matter.

I. Mathematical analysis of a collapsing halo of Dark Matter

A. Modeling a collapsing halo

As usual in physics when we try to understand a complicated phenomenon, we try to find some hypothesis we could make to make our problem simpler, but that would keep it realistic.
In our case, we will make the following assumptions :

* Non-gravitational forces are negligible
* Dark matter has no initial velocity, except the one due to expansion.
* Dark matter has no angular momentum and the collapse is purely radial.

Our first assumption can be considered as realistic : most scientists that work on dark matter problem consider that dark matter only, or mostly, interacts through gravitational interaction. Our second assumption is based on the hypothesis that dark matter is cold (cold dark matter model**[7][8]**), which means that it's made up of slowly moving particles.Our last assumption, assuming that the collapse is purely radial, is not very realistic, but we should still get a good idea of what happens during and after the collapse, keeping in mind that if we want to make a more realistic calculation, we should get rid of that assumption first. In the conclusion we will discuss on the effect that could have a non-radial collapse (still with no global angular momentum).

The spherical model is a very simple one but can still give us a good approximation of what happens in reality.
Moreover, we can easily see that it is much easier to make a simulation considering *spherical layers* and not particles, that's why we chose to represent the dark matter halo as a superposition of N layers, which makes it far easier to compute and can give very good results if N is big enough.

Another approximation we are making in our program is making a time step small enough so we can consider that layers do not cross ; in other words the approximation we are making is considering that the inner mass (ie the mass within a given layer) has only varied sightly during the time step (between t and t+dt).

B. Collapse in an expanding universe

We are considering the collapse of a cold dark matter halo in an expanding universe using Newtonian Dynamics **[9]**. Our assumption that neutralinos have very low initial velocity means that $v_{init} \ll HR_0$, which means that the structure we are talking about cannot be too small (if R_0 is too small, then expansion has very little influence and our assumption becomes unrealistic).
Considering the gravitational interaction :

$$\boxed{\frac{d^2 r_i}{dt^2} = \frac{-1}{r_i^2} GM_i}$$

where r_i is the distance from the layer to the center of the halo, M_i is the mass within the considered layer and making some algebra, we find the relation $v_i^2 = \frac{2\mathrm{GM}_i}{r_i} + C$ where $\boxed{C = -2\frac{GM_i}{R_{0i}} + v_{oi}^2}$ and represents twice a layer total energy, and where $R_{0\mathrm{i}}$ and v_{oi} are defined as the position and the speed of layer i at some initial time.

The solution of the differential equations varies whether C is positive or negative. If C<0, it means that the layer hasn't enough energy to escape from the gravitational attraction of the halo and will collapse again ; if C>0, the layer can escape from the halo, so it won't collapse any more.

i) Case C<0

In that case, the solution of the differential equation can be written parametrically :

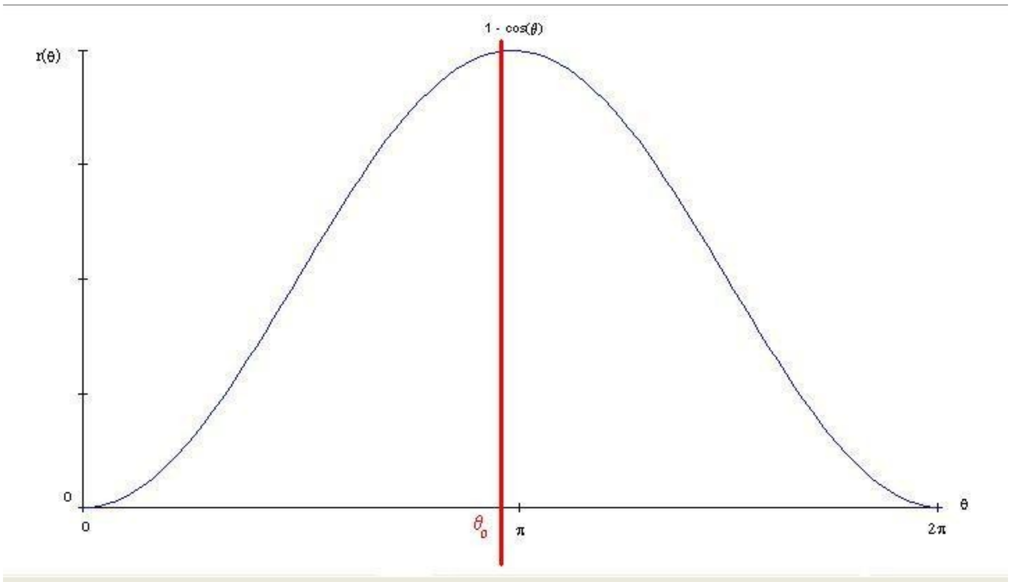
$$r_i = A_i (1 - \cos(\theta_i)) \;\; ; \;\; t = B(\theta_i - \sin(\theta_i)).$$

with $B = \sqrt{\frac{A^3}{GM_i}}$

We now have to make the calculation of the initial conditions, which means finding θ_0 and A for each layer that we will then use to find the new position and speed of the layer.
for t=0 we get $R_{0\mathrm{i}} = A_i (1 - \cos(\theta_{0\mathrm{i}}))$; $v_{0\mathrm{i}} = A_i \dot{\theta}_{0\mathrm{i}} \sin(\theta_{0\mathrm{i}}) = HR_{0\mathrm{i}}$.
So we immediately find $\theta_{0\mathrm{i}} = 1 - \frac{R_{0\mathrm{i}}}{A_i}$.
As we have $\cos^2(\theta_i) = (1 - \frac{R_{0\mathrm{i}}}{A_i})^2$ and $\sin^2(\theta_i) = \frac{H^2 R_{0\mathrm{i}}^4}{GMA_i}$ we find $A_i = \frac{R_{0\mathrm{i}}}{2 - \frac{H^2 R_{0\mathrm{i}}^3}{GM_i}}$
At the beginning all the layers have C<0 (if some layers already have C>0 then they don't collapse so they are not interesting at all). We can evaluate the time when the halo should collapse ; the collapse takes place when $r = 0$ which

means $\theta=2\pi-\theta_{0i}\approx\pi$ [1]so we get $\tau=\pi B=\pi\sqrt{\frac{A^3}{GM_i}}$.
We have made a separate calculation for the inner layers (which had their mass multiplied by two), and for the outer ones, as layers of different mass don't have the same collapse time (see Table in part II.c).

The reason why we take $2\pi-\theta_{0i}$ is because θ indicates the difference with θ_{0i} , which is already ~ π as we are simulating a collapsing halo, and not something expanding *then* collapsing :



As we begin at $\theta=\theta_0$, we only have to make π (approximately) to reach the collapse.

ii) Case C>0
In that case, the solution of the differential equation can be written parametrically :

$$r_i=A(ch(\theta_i)-1)\ ;\ t=B(sh(\theta_i)-\theta_i).$$

for t=0 we have $r_{0i}=A(ch(\theta_{0i})-1)\ ; v_{0i}=A\dot{\theta}_{0i}sh(\theta_{0i})=HR_{0i}$.

We get $\theta_{0i}=acosh(\frac{R_{0i}}{A_i}+1)$ and we find $A_i=\frac{R_{0i}}{\frac{H^2R_{0i}^3}{GM_i}-2}$

At the beginning all the layers have a negative total energy (which means they all have C<0).

During the collapse, energy is transferred from one layer to another. That's why at the end of the collapse some layers have a positive energy, which means that their velocity is higher than escape velocity (so they will escape) ; but some are still going to have C<0 so after the first collapse we will have remaining layers that can collapse again, which means that we should be able to see the formation of many (more than one) caustics.

However, the caustics should become weaker and weaker with time, as each time there is a collapse some layers escape, there are always fewer layers remaining.

1 In every simulation we get $3.10\leq\theta_{0i}\leq3.20$, that's why we can approximate $\theta_{0i}\approx\pi$

II. Dark matter halo and formation of caustic rings : Computing the collapse

Present observational data give very poor constraints on how dark matter is distributed in small-scale structures. Multiple simulations is that if dark matter matter is cold (cold dark matter model, CDM), then it has to form clumps on every scale, from the largest structures of the universe to the smaller ones (earth-mass structures).

As we are working on cosmic scales, the unit system we usually work with is not appropriate and we prefer using units that are commonly used in astrophysics : parsecs, Myear and Solar Masses (M_o).

A. Using a more natural unit system

In the international unit system we have :

$G=6.67\times10^{-11} m^3 kg^{-1} s^{-2}$

$H_0=70\, km\, s^{-1}\, Mpc^{-1}$

As we have

$1\text{m}=3\times10^{-17}\, pc\,;\, 1\text{s}=3\times10^{-14}\, Myr\,;\, 1\text{kg}=5\times10^{-31}\, M_o$.

we easily get :

$G=3.2\times10^{-3}\, pc^3 M_o^{-1} Myr^{-2}$ and $H_0=10^{-4}\, pc\, Myr^{-1}\, pc^{-1}$.

H_0 value is the one we get today. In the simulations, we are going to suppose that when dark matter haloes formed it was approximately 10^3 times greater (which means that they formed just after the CMB), so we will take $H=10^{-1}\, pc\, Myr^{-1}\, pc^{-1}$ which should be a more realistic value for H.

B. Numerical resolution of the collapse

The first thing the program, made in C++ using Root, has to do is to create the layers and give the same mass to each of them (it just divides the total mass of the halo, which is an input, by the number of layers we ask it to create).
As we will see in *part C*, caustics form when the halo is heterogeneous, that's why we then make the program multiply by two the mass of all the layers situated below a certain radius. Another initial condition is that the Universe is in expansion, so we have to give an initial speed to every layers according to their distance to the center. Now that we have the initial radius and speed of each mass, we can compute θ_0 and C. In the program $R_{0\text{i}}$ and v_{oi} are defined as the position and the speed of layer i at time t-dt, because as the calculation of C has to be made at every step, the input has to be the last position and speed of the layer. According to the sign of C, we know which equations we have to describe its motion and we can find its new radius and new speed.

The architecture of the program is based on the fact that we are making very small time steps. For every step, we compute the new distance between a given layer and the center, its speed, and its total energy C (using its new radius and speed, that's why C changes with time). We then check if C is positive or negative, so we know which formula we need to use to compute its motion.

C. Simulation results

Before trying to simulate real structures, we simulated a simple and unrealistic structure to try to understand what conditions need to be required for the formation of caustics :

. The first simulation was a simple completely homogeneous halo : did caustics form ?

. If they don't form, what happens if the inner layers are more dense than the outer ones (which is a more realistic simulation) : do caustics form ?

. If caustic now form, what is the shape of the density peak ? How does it evolve with time ?

For our first simulation we are going to considerate a homogeneous halo (total mass $1.125\times10^3 M_o$, initial radius of 1.0pc). We get **Figure 1 :** when all the layers have the same mass, and the halo is homogeneous, there is no warping in phase space. All the layers behave the same way and no caustics form. Therefore we decided to work on a heterogeneous distribution, keeping all the other parameters : the layers which were initially situated below $\frac{R_0}{2}$ had their mass multiplied by two. The results are visible on **Figure 2** : in phase space, we could see the caustic rings forming. All the following simulations will be made using that heterogeneous distribution, as it seems to be a necessary condition for the formation of caustics. On **Figure 3**, we clearly see that the warping in phase space leads to a realistic density peak : approximately 75% of the mass is concentrated in 20% of the initial distance.
On **Figure 4** is shown the evolution of caustic rings with time. We see that caustics get weaker and weaker as predicted analytically.

On the following table, we are comparing collapse time τ given analytically with the collapse time given when using the simulation. As we can see, the theoretical results are very close to the computed ones :

	Inner layers (Ro=0.5pc)	Outer layers (Ro=1.0pc)
Theoretical time for collapse (Myr)	0.45	0.58
Simulation (Myr)	~0.46	~0.57

On **Figure 5** we can clearly see when caustics form and how the annihilation signal varies with time. Even though it is not calibrated (figure 5 just shows $\rho^2 = f(t)$), it gives us an idea on how powerful the signal be when the halo collapse and how caustic rings makes peaks).

We can now try to simulate more realistic structures.

**Simulating the collapse of a galactic-scale halo.*

For $M = 1.0 \times 10^{10} M_o ; R_0 = 1.0\, Mpc$ we get **Figure 6** ; for a total mass of $1.0 \times 10^{14} M_o$, and an initial radius of $100\, Mpc$ we get **Figure 7.**

The formation of the caustics are visible on the graph, however what we can deduce from this graph is that except the collapse and the first two caustics, the other ones are very difficult to distinguish from the background.

**Simulating the collapse of smaller structure : earth-mass dark matter halos*

For $M = 10^{-6} M_o$ and $R_0 = 10^{-11}\, pc$ we get **Figure 8**, and we also did the simulation for $M = 10^{-6} M_o$ and $R_0 = 10^{-9}\, pc$ but the radius was considered to be to small for that kind of structure and didn't agree with our no initial velocity assumption.

It seems that in every case (if inner mass is greater than outer mass), we have formation of caustic rings.

III. Neutralino annihilation signal

The Standard Model of Particle Physics has no particle that could be a serious candidate to be the major component of Dark Matter. The only way to find a good candidate is to go beyond the Standard Model. Supersymmetry **[10]** assumes symmetry between the fermions an the bosons and predicts the existence of new particles ; one of them, the neutralino, could be a very good candidate for dark matter, which should be stable if R-parity is verified and if it is the lightest superparticle, a scenario which is supported by many authors. Neutralinos might be indirectly detected by their annihilation, as they should produce high energy gammas, neutrinos, positrons **[11]** and many other standard model particles that would provide ideal observational results.

The flux of dark matter annihilation products is proportional to the number of annihilations per unit time, per unit volume : $N \propto \rho^2$. The neutralino annihilation signal is therefore given by the following formula :

$$\boxed{N = \kappa \frac{\sigma_{xx} v}{m_{xx}^2} \rho^2}$$

where[2] $\sigma_{xx} v \approx 10^{-29} cm^3 s^{-1}$ is the neutralino annihilation cross-section, $m_{xx} \approx 100 GeV/c^2$ its supposed mass and where κ indicates the probability that the neutralino annihilation gives photons.
Knowing that $1 GeV/c^2 = 1.783 \times 10^{-27} kg$, we get $m_{xx} \approx 1.783 \times 10^{-25} kg = 8.9 \times 10^{-56} M_o$
and $\sigma_{xx} v \approx 10^{-35} m^3 s^{-1} = 8.1 \times 10^{-72} pc^3 Myr^{-1}$.

Taking $\kappa = 1$ (which means we assume that every annihilation gives a photon), we get a good approximation with :

$$N \approx 10^{41} \rho^2 cm^{-3} \cdot s^{-1}$$

Results for different structures

The boost factor is defined as $\frac{\langle \rho \rangle^2}{\langle \rho^2 \rangle}$ so it explicitly gives the coefficient by which the intensity was multiplied due to the caustics. If there were no caustics, the boost factor would be 1.

	Type	$\rho^2 (M_o^2 \cdot pc^{-6})$	N $(cm^{-3} \cdot s^{-1})$	**Boost Factor**
Total Mass : $1.0 \times 10^{14} M_o$ Initial Radius : $100 Mpc$	Galaxy cluster (fig7)	8.5×10^{14}	~ 10^{-6}	**Collapse time: 85000** **First peak : 2500** **Second peak : 500** **Third peak : 30** **Fourth peak : 2**
Total Mass : $1.0 \times 10^{10} M_o$ Initial Radius : $1.0 Mpc$	Galaxy (fig6)	1.7×10^{11}	10^{-9}	**Collapse time: 17000** **First peak : 250** **Second peak : 110** **Third peak : 60** **Fourth peak : 2.5**
Total Mass : $M = 10^{-6} M_o$ Initial Radius : $R_0 = 10^{-9} pc$	Earth-like structure, 100time larger	2.5×10^8	~ 10^{-12}	**Collapse time: 80000** **First peak : 1200** **Second peak : 500** **Third peak : 150** **Fourth peak : 3**
Total Mass : $M = 10^{-6} M_o$ Initial Radius : $R_0 = 10^{-11} pc$	Earth-like structure (fig8)	10^{13}	10^{-7}	**Collapse time: 58000** **First peak : 2400** **Second peak : 1700** **Third peak : 300** **Fourth peak : 250**

Consequences of the caustics

In every simulation we have made, we noticed that only first two or three caustics had a significant impact on the annihilation signal, as it can become nearly 3000 times more powerful than the background average signal.

However, the contribution of all the other caustics seemed to be *extremely low*, and we should not be able to distinguish them from background signal (the boost factor is nearly 1 after the fourth peak in every simulation, except the last one but its radius is not realistic-it is too small- so it doesn't really agree with our no initial velocity assumption).

2 The most recent measurements (Cern) gives a lower limit of 32 GeV/c² for the neutralino mass

Conclusion

In this document, I tried to understand how caustic rings can form and their possible implication in astrophysics and cosmology.

To see the formation of caustic rings, we had to make several assumptions : we worked in a very simplified framework, using a spherical halo of dark matter where all the layers had no initial velocity, except the one due to expansion. We also made the assumption that the halo had no angular momentum, which is not a realistic assumption but simplifies a lot the simulation.

However, I think we can still trust the results obtained : spherical symmetry is often used to get an idea of what happens and it gives pretty good approximation of reality, no initial velocity can be considered as a fair assumption for large-enough structures, however for earth-mass structures this hypothesis may gyve only a poor approximation of reality. If the halo had an angular momentum, it would probably only have reduced the intensity of the peaks, but would certainly not go against their formation **[13]**. In our simulation of a purely radial collapse, all the layers were falling on the same point (the center of the halo). However, if the layers have an angular momentum, they won't all be falling on the same point so we won't have a so intense intensity peak at the collapse time as we currently have.

Even though my work was done in a very simplified framework, we obtained some interesting results concerning the evolution of the neutralino annihilation signal. We saw that the first caustics may help us in detecting dark matter, however the neutralino annihilation signal weakens very fast and therefore quickly becomes impossible to distinguish from the background signal.

Acknowledgments

I would like to thank all the researchers and all the personnel that work in the LAPTH and the LAPP for their great reception and for all the documents and material they put at my disposition for my work.
I would like to thank in particular Richard Taillet, my traineeship supervisor, that gave me many useful indications that helped me work in the best conditions, helping me on the optimization of the C++ program and giving me many advice on how to write a good scientific paper.
I really enjoyed working on that subject, which made me learn a lot on what could be dark matter and how it may be distributed, and helped me better understand what the work of a researcher consist in.

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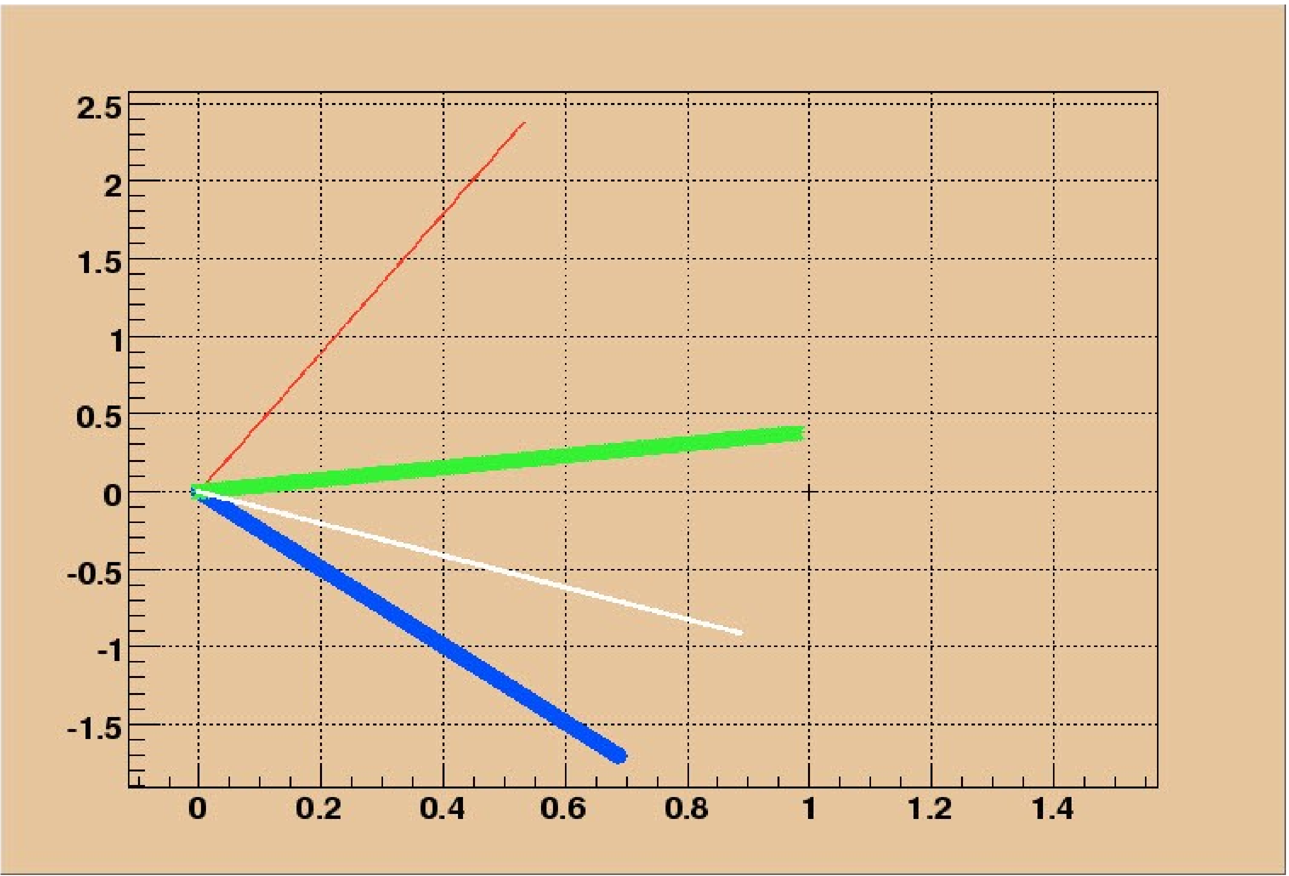


Figure 1 :All the layers have the same mass, no caustic rings.

Phase space diagram $(r, \dot{r})$

Total Mass : = $1.125 \times 10^3 M_o$; Initial Halo Radius = 1.0pc ; 1000layers

Color indicates given times : Blue, Green, Red and White.

We see that no caustics appear, this is due to the fact that all the layers behave the same way.

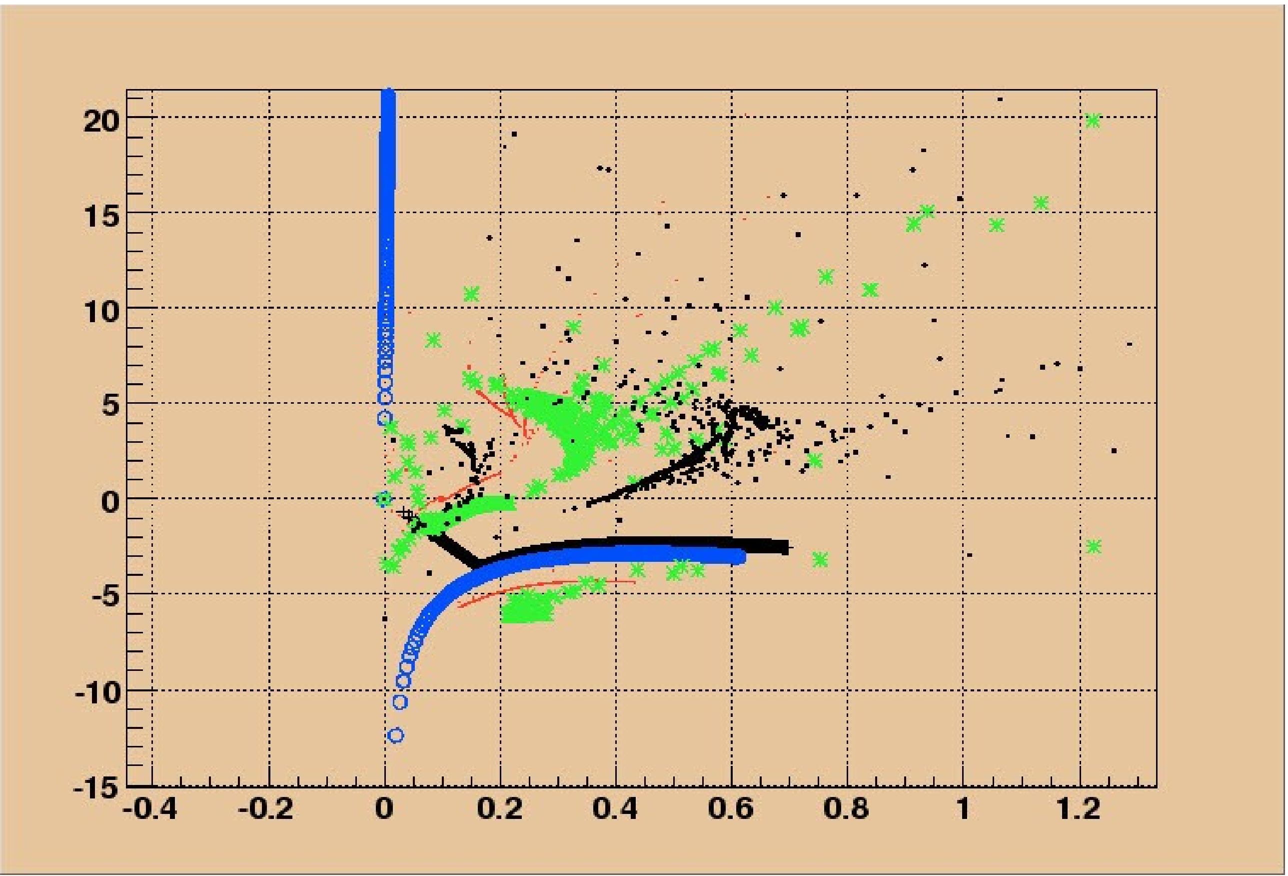


Figure 2 : Formation of caustic rings.

Phase space diagram $(r, \dot{r})$. Dots indicate the layers.

Total Mass : = $1.125 \times 10^3 M_o$; Initial Halo Radius = 1.0pc ; 1000layers

Color indicates given times : Black, Blue, Green, Red and Black.

For the first black line, the collapse has just begun.
Some layers have already collapsed for the blue line.
Red one shows the first caustic ring at R=0.25pc
Green one shows the same caustic around R=0.35pc, and shows another caustic in formation around R=0.05pc.
The second black line shows clearly the two caustic rings around R=0.6pc and R=0.15pc.

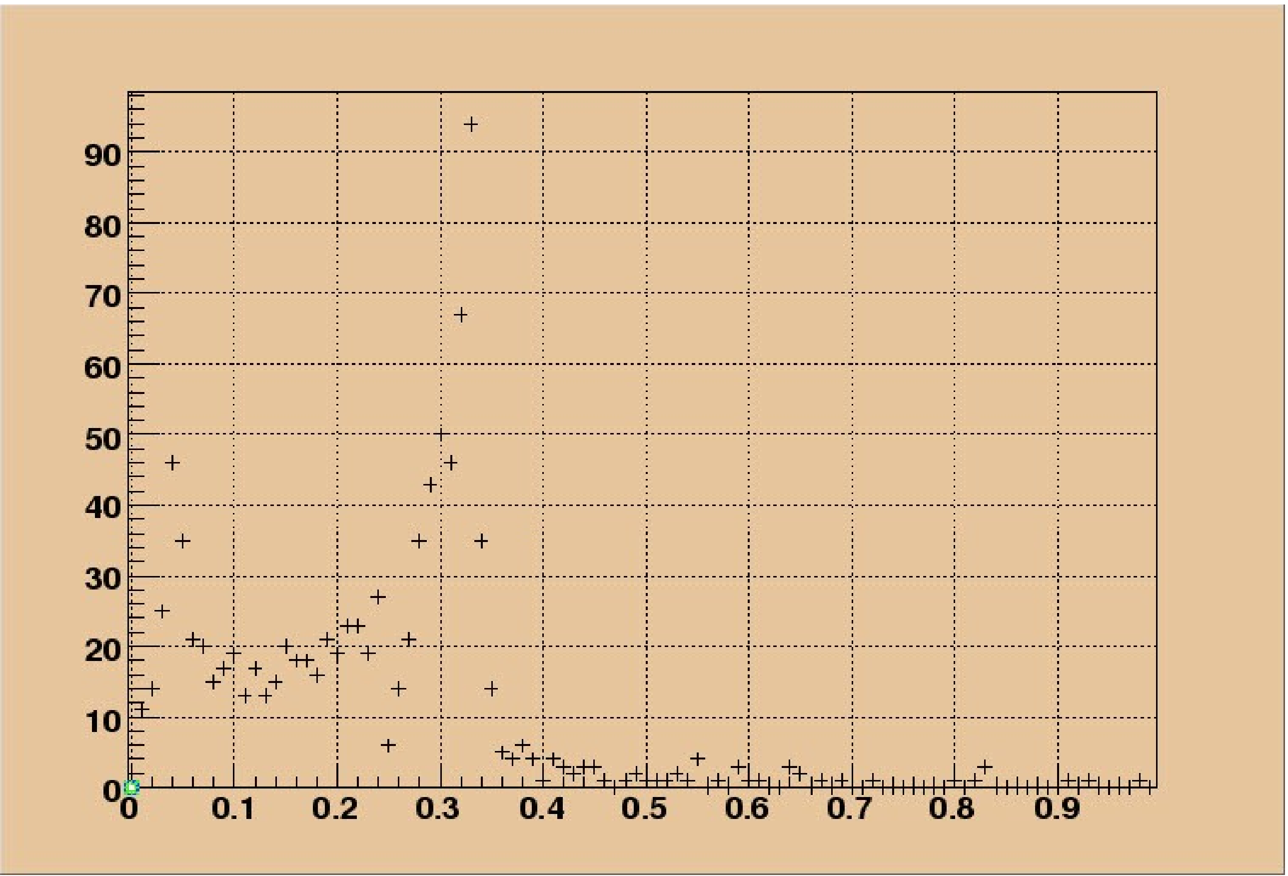


Figure 3 : Evolution of density (number of layers) with radii at given time (collapse has already begun and caustic rings are clearly visible).

Total Mass : = $1.125 \times 10^3 M_o$; Initial Halo Radius = 1.0pc ; 500 layers

We clearly see 2 density peaks at R=0.02pc and R=0.32pc. Those density peaks are the caustic rings.

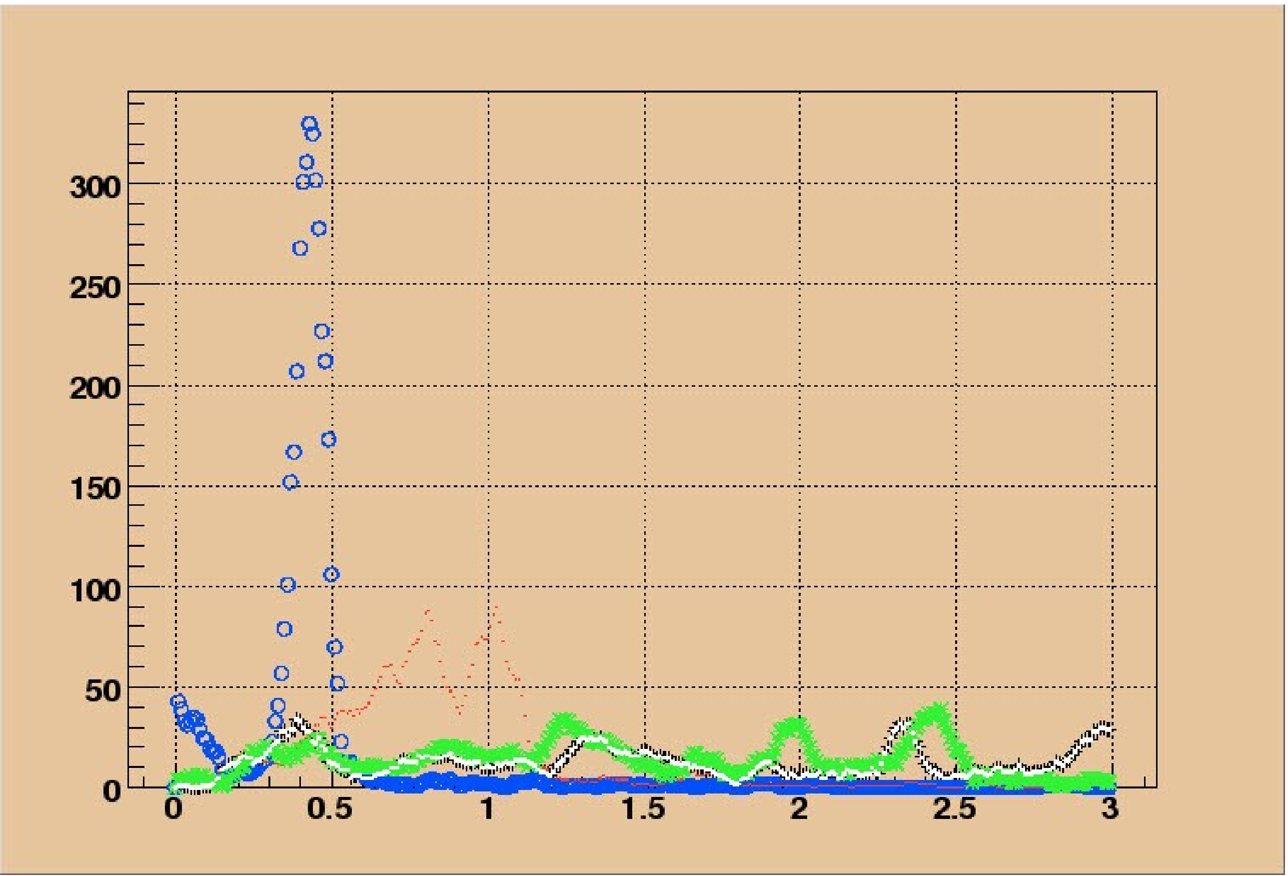


Figure 4 : Evolution of caustic rings (number of layers) with radius.
Colors represent evolution with time : Blue, Red, Green, White.

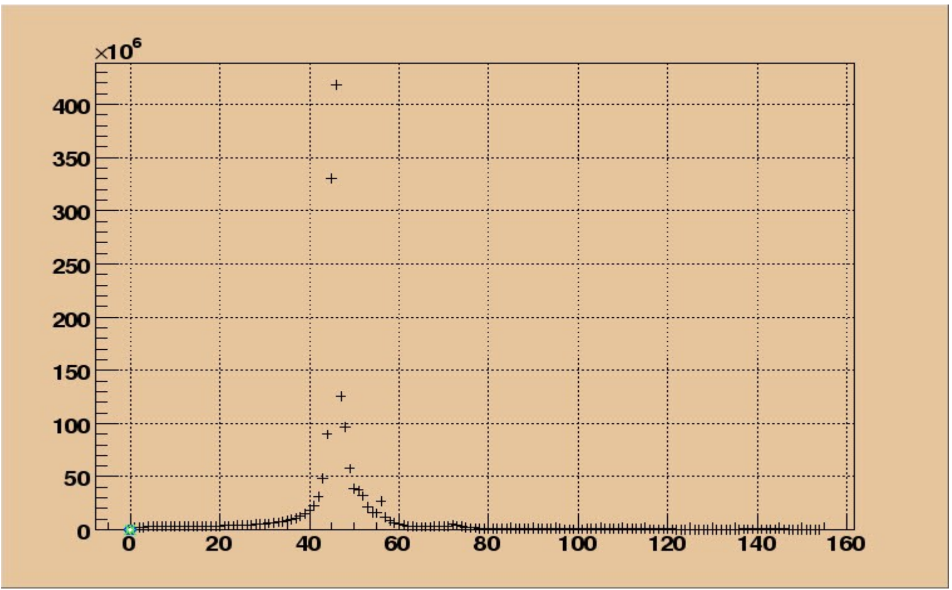
Total Mass : = $1.125\times10^3 M_o$; Initial Halo Radius = 1.0pc ; 500 layers

Blue : we clearly see the first peak (the first caustic) around R=0.4pc, and one in formation.

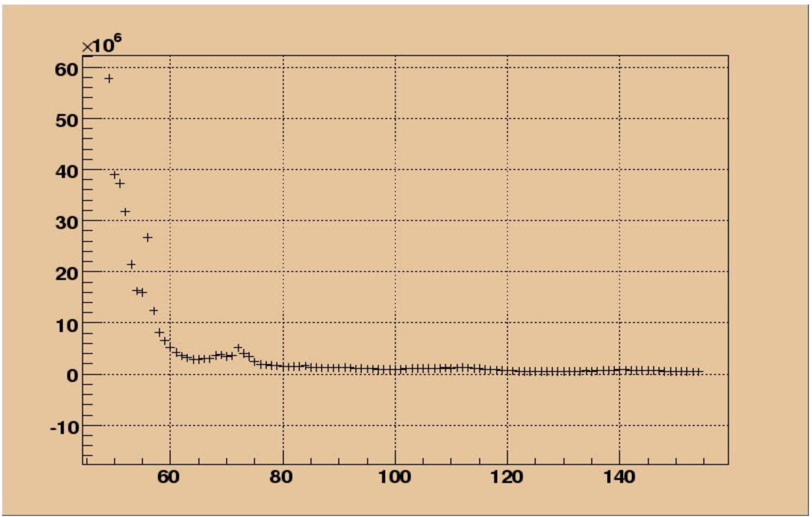
Red : two peaks around R=1.1pc and R=0.8pc, and antoher one ine formation around R=0.3pc.

Green : 4 caustics around R=2.4pc, R=2.0pc, R=1.25pc, and R=0.4pc.

White : 4 distinguishable peaks around R=3.0pc, R=2.3pc, R=1.3pc, and R=0.4pc.



Zoom :



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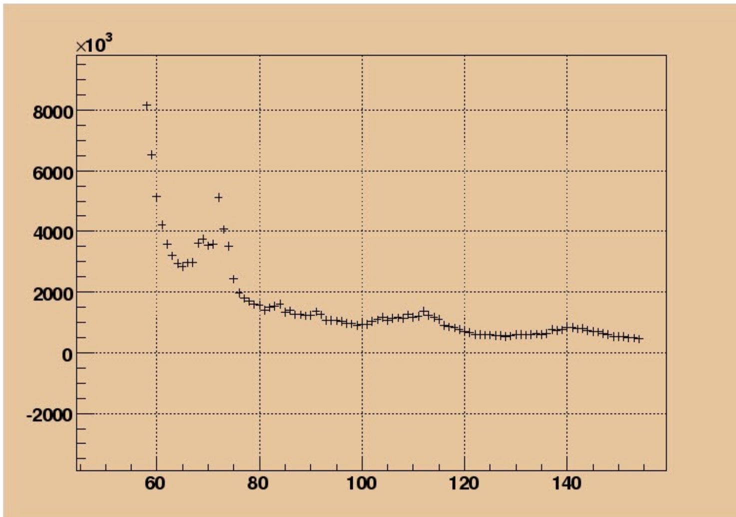
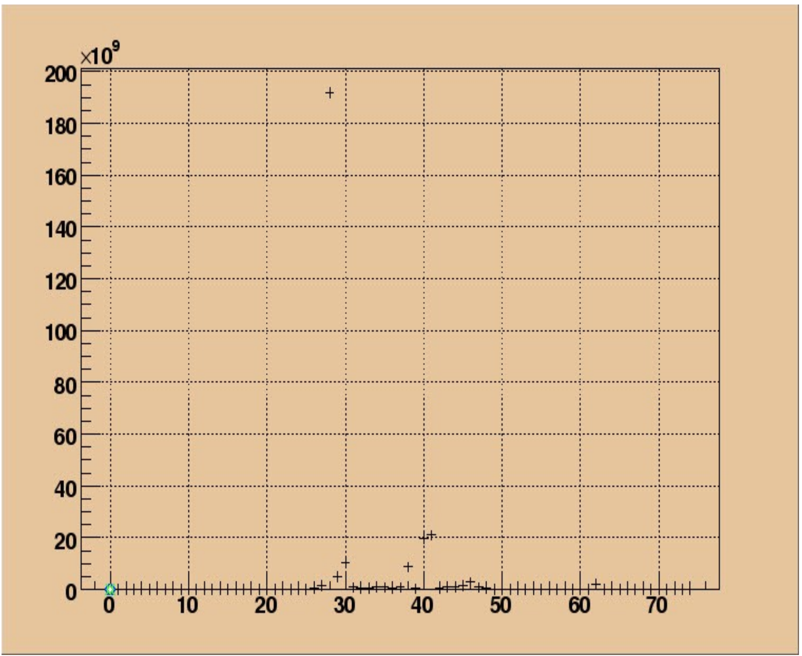


Figure 5 : Intensity of annihilation signal over time($\rho^2 = f(t)$).

Total Mass : = $1.125\times10^3 M_o$; Initial Halo Radius = 1.0pc ; 500 layers

This figure shows the non-calibrated number of annihilation that should take place over time. As predicted analytically, we clearly see the collapse at t=0.46. But what is more interesting in this figure is to see what happens after the collapse. We clearly see that the formation of the caustics is indicated by the presence of a bump on the graph (local maximum), which means a more intense signal.

The peaks are situated around t=0.57, t=0.71, t=0.82, t=0.91, t=1.15, t=1.40 Myr



Zoom :

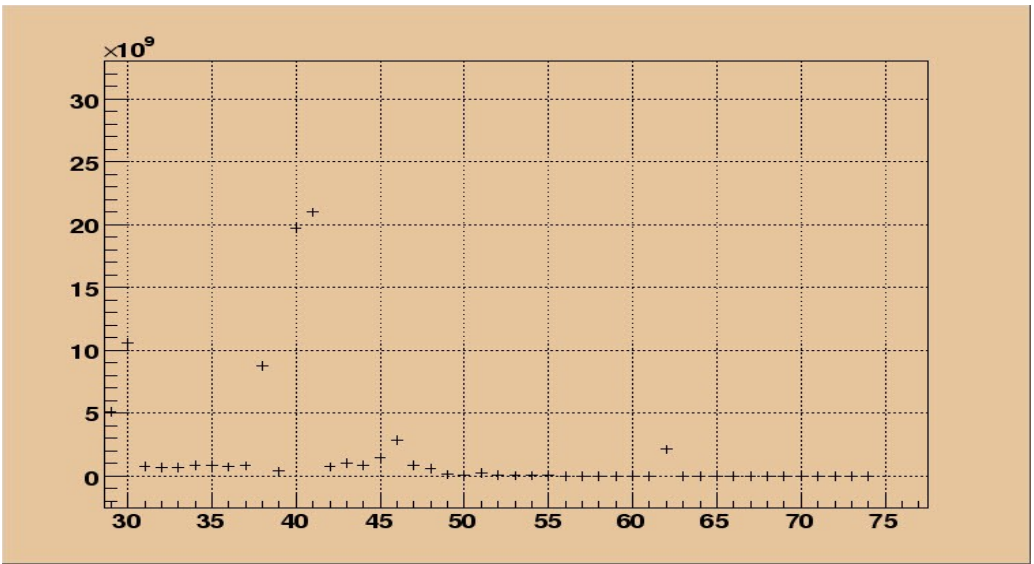
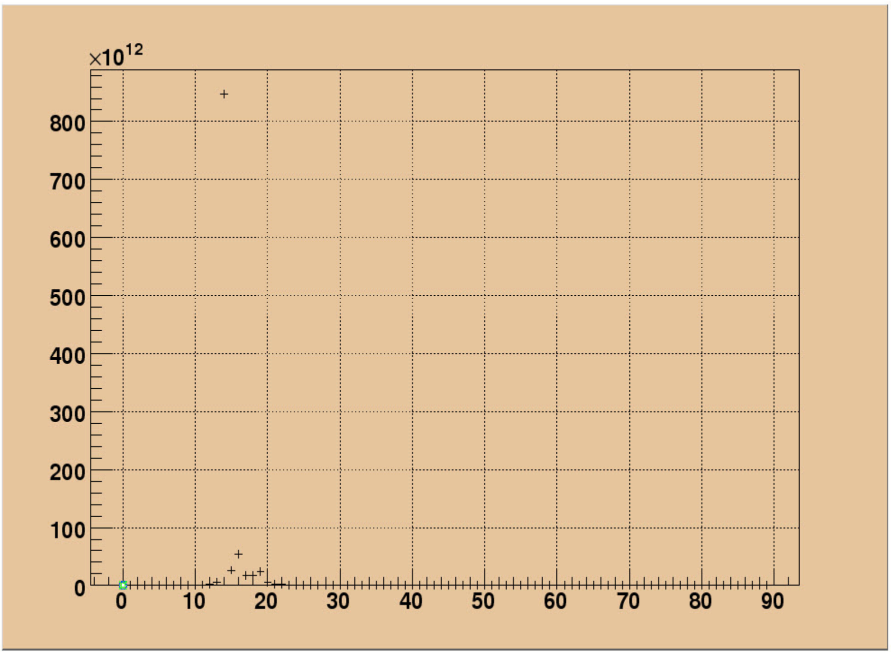


Figure 6 :Intensity of annihilation signal over time for a galactic-scale halo ($\rho^2 = f(t)$).

Total Mass : $1.0 \times 10^{10} M_o$, Initial Radius : $1.0\, Mpc$, 500 layers

Again we see that the formation of the caustics are clearly visible on the graph.



Zoom :

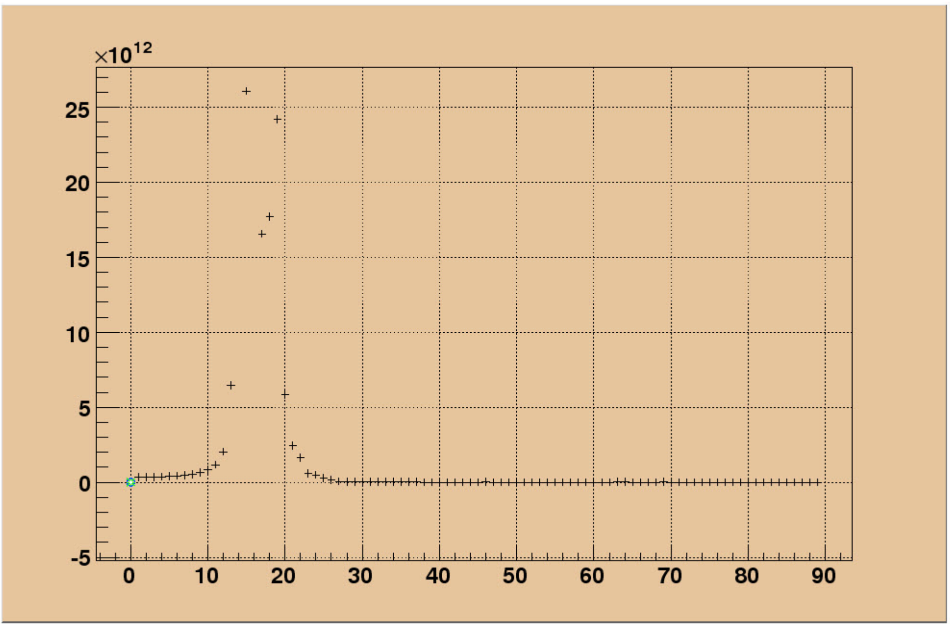
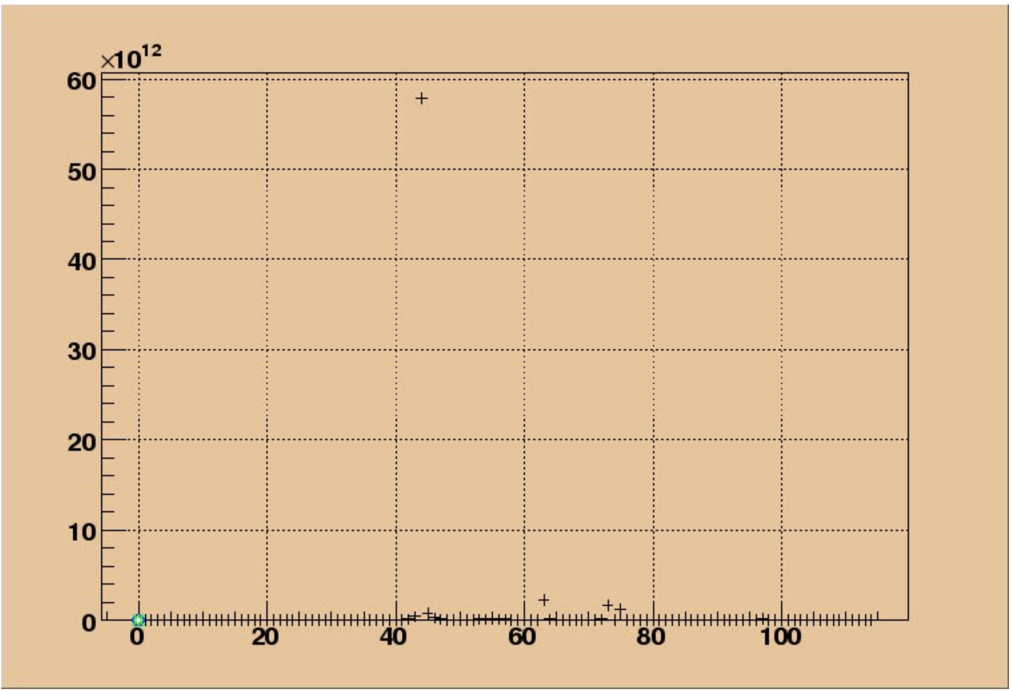


Figure 7 :Intensity of annihilation signal over time for a very large-scale halo ($\rho^2 = f(t)$).

Total Mass : $1.0 \times 10^{14} M_o$, Initial Radius : $100\,Mpc$, 5000 layers

Again we see that the formation of the first caustics are visible but very quickly the next peaks are very low compared to the background emission.



Zoom :

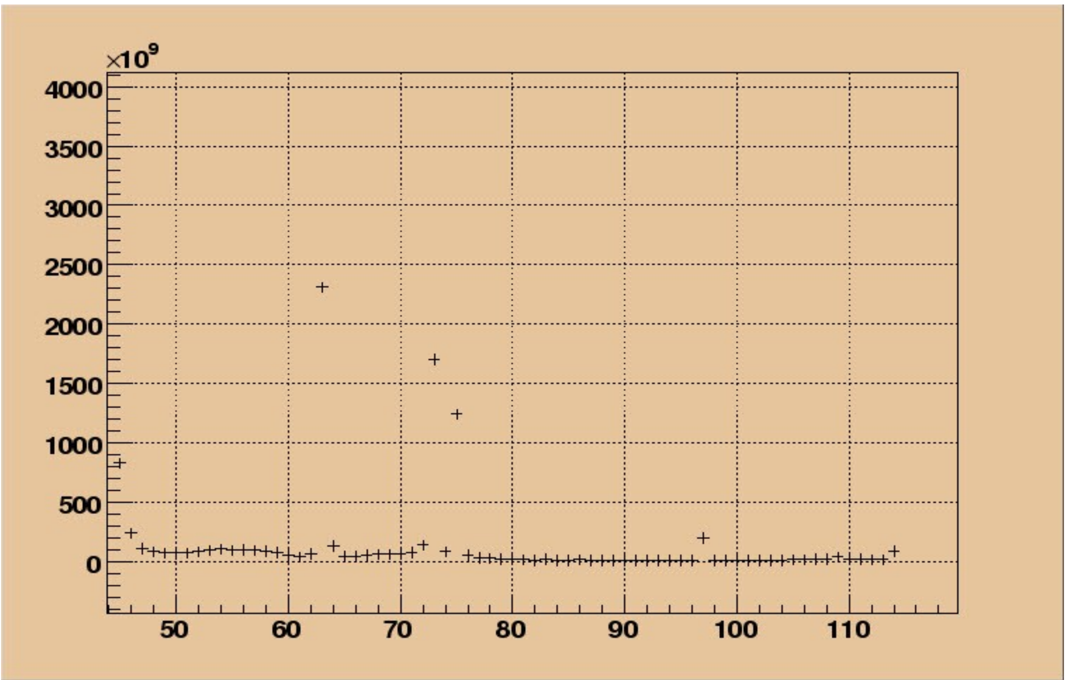


Figure 8 :Intensity of annihilation signal over time for a earth-like halo ($\rho^2 = f(t)$).

Total Mass : $1.0\times10^{-6} M_o$, Initial Radius : $10^{-11} pc$, 500 layers

Again we see that the formation of the first caustics are clearly visible on the graph, but the following ones are at the background-intensity level.