Physics beyond the Standard Model : Effective Lagrangian Approach to New Physics

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1 Introduction

The Standard Model of Particles is a theory of three of the four known fundamental interactions ; it ties together in a coherent framework the electroweak theory -electromagnetic and weak interactions-, quantum chromodynamics -strong interactions- and all fundamental particles that take part in these interactions.

Despite its remarkable success in the last few decades and strong agreement with experiments, particle physicists know that this may not be the ultimate theory of funamental interactions : it lacks any good explanation for neutrino osciallations, it suffers from what is know as a "hierarchy problem" (11 ordrers of magnitude between the weak scale and Planck scale), and it does not include gravitation.
Another well-known problem in the standard model arises when computing the $WW \to WW$ scattering amplitude : unitarity in the Standard Model is lost due to the longitudinal polarization of the W boson which is proportional to its energy and can therefore grow indefintely. The Higgs mechanism however can solve this problem if the Higgs boson is found with a mass around 100 - 200 GeV (we should know this very soon, thanks to the LHC).

There are two different paths to study physics beyond the Standard Model. One can choose to work through a *model dependant approach* : this is for exemple done with the SuperSymmetric or the Technicolor approaches ; a whole new framework, new particles, new types of interactions. All predictions are made within that new framework and of course it has to reproduce all known datas accurately. Or one can choose a *model independent approach* for instance by adding extra-terms to the Standard Model Lagrangian, therefore making it an effective (non-renormalizable) Lagrangian. Loss of renormalizability is not a problem here : we are only interested in looking what kind of phenomenom could happen due to these extra terms (low energy limit of a very high energy new physics). This is the path I followed during my internship at the CERN.

My work was roughly divided into two parts : one part was more theoretical and was about understanding the physics contained in the new operators we were adding, finding the relations that could link some of them together and discussing their possible impact on electroweak baryogenesis. The other part was computational : how to compute scattering amplitudes for non renormalizable operators ? Can any other physical conscequences (mass corrections,...) be extracted numerically ?

2 Theoretical approach

2.1 Standard model Lagrangian (electroweak-higgs sector)

The Standard Model can be divided into three distinct pieces : the electroweak sector, the strong sector, and the higgs sector. Being interested in the effect of electroweak transition, we are not interested in the strong sector that is totally decoupled from the electroweak interaction at this energy scale, and therefore we only need to consider the electroweak-higgs sector.
The electoweak-higgs sector of the standard model lagrangian can be written as follows:

$$L_{SM_{EW}} = -\frac{1}{4}W^a_{\mu\nu}W_a^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B_{\mu\nu} + (D_\mu H^\dagger)(D^\mu H) - \mu^2 H^\dagger H - \lambda(H^\dagger H)^2 \tag{1}$$

where the field strengths and the covariant derivative are defined as :

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon_{abc}W^b_\mu W^c_\nu$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$D_\mu = \partial_\mu + ig\frac{\sigma^a}{2}W^a_\mu + ig' B_\mu$$

and with

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \pm iW^2_\mu)$$

$$Z_\mu = cos(\theta_w)W^3_\mu - sin(\theta_w)B_\mu \quad ; \quad A_\mu = sin(\theta_w)W^3_\mu + cos(\theta_w)B_\mu$$

The effective lagrangian we are are willing to build can only contain these fields. Of course, it would possible to construct theories with new interactions, new particles and/or different symmetries, but this is *not* what we are trying to do here.
Here, the approach it to consider that all experimental evidence is against such kind of "new physics", at least up to the TeV scale, which is the energy scale we are interested in.

Variations of this lagrangian with respect to the fields gives us the Standard model equations of motion :

$$D^\nu W^a_{\nu\mu} + g(iH^\dagger \frac{\sigma^a}{2} D_\mu H + h.c.) = 0 \tag{2}$$

$$\partial^\nu B_{\nu\mu} + g'(iH^\dagger D_\mu H + h.c.) = 0 \tag{3}$$

2.2 Effective extension of the Standard model

Let's suppose that at energies above the SM energy scale ($> TeV$), there is a bigger symmetry group that breaks around a few TeV to give the following structure : $SM(SU(3) \otimes SU(2) \otimes U(1)) + E_{SM} + E_{\underline{SM}}$, that is the Standard Model structure, extra terms that also have the SM gauge symmetries, and extra terms that don't.
Let's assume that physics at the TEV scale still has the symmetries we know, i.e let's consider E_{SM} only.

$$L_{eff} = L_{SM_{EW}} + \sum_{i=1} \frac{c_{\alpha}^{(i+4)}}{\Lambda^i} O_{\alpha}^{(i+4)} \tag{4}$$

As previously stated, one of the constraint we imposed is that our new effective lagrangian must respect the various Standard Model symmetries and conservation laws (flavor conservation,...): it turns out that only even dimension operators are allowed[1]. Therefore we will only consider i=2 (i.e. dimension 6 operators $O_{\alpha}^{(6)}$), as higher order even operators would be supressed by a factor of Λ^{i+4}, Λ -being of the order of a few TeV-. Well, it turns out that just for dimension 6 operators satistifying the SM symetries we are already talking about 80 terms (cf Buchmljller and Wyler paper) ! Therefore, our approach will be to deal only with a few of those extra non renormalizable terms.

The dimension 6 operators that we will be considering here are the following ones :

$$O_s = (H^\dagger \sigma^a H) W^a_{\mu\nu} B^{\mu\nu} \ , \ O_T = |H^\dagger D_\mu H|^2$$

$$O_W = (D_\rho W^a_{\mu\nu})^2 \ , \ O_Y = (\partial_\rho B_{\mu\nu})^2$$

$$O_{3W} = \epsilon^{abc} W^{\mu a}_{\nu} W^{\nu b}_{\tau} W^{\tau c}_{\mu} \ , \ O_{DH} = H^\dagger H |D_\mu H|^2$$

$$O_{BH} = (B_{\mu\nu})^2 H^\dagger H \ , \ O_{BDH} = i B_{\mu\nu} D^\mu H^\dagger D^\nu H$$

$$O_{WH} = (W^a_{\mu\nu})^2 H^\dagger H \ , \ O_{WDH} = i W^a_{\mu\nu} D^\mu H^\dagger \sigma^a D^\nu H$$

$$O_{\partial H} = (\partial (H^\dagger H))^2 \ , \ O_{HDH} = (H^\dagger D_\mu H)^2$$

Some of these operators lead to very interesting new physics ; for example, O_S indicates a mixing between the Z-boson and the photon. Also, note that electroweak precistion tests allow us to put rather strong constraints on the most c_α coefficients.

[1]One dimension-5 operator respects the SM global symmetry but violates lepton number

2.2.1 Dimension 6 operators interdependence

The first thing that should be noticed is that some of these operators are not independant. For instance, by multiplying the SM equations of motion respectively by $(iH^\dagger\sigma^a D_\mu H + h.c.)$ and by $(iH^\dagger D_\mu H)$ we get (see Appendix A_1), by using Fierz transformations,

$$6gO_{DH} - g'O_S - 4O_{WDH} - gO_{WH} = 0$$

$$4g'O_T - gO_S - 4O_{BDH} - g'O_{BH} + 2g'ODH = 0$$

Also, one can start from the Bianchi identites of the strength fields and the SM eqquations of motion to get (see Appendix A_2)

$$O_W + 2gO_{3W} + 3g^2O_{DH} = 0$$

$$O_Y - 2g'^2O_T - g'^2O_{DH} = 0$$

One can also use integration by parts on those dimension-6 operators to get (see Appendix A_3) :

$$O_{\partial H} + O_{DH} = 0$$

$$O_T + O_{DH} + O_{HDH} = 0$$

2.3 Triple Gauge Boson Coupling

The new dimension 6 operators can induce new interaction between particles. In particular, interactions between the W and the Z and the W and the A (photon) lead to anomolous triple gauge coupling, in other words deviations from the usual standard model interaction vertex.

This anomaly can be parametrized and written as follows:

$$L_{eff}^{WWV} = g_{WWV}[g_1^V V^\mu(W_{\mu\nu}^- W^{+\nu} - W_{\mu\nu}^+ W^{-\nu}) + \kappa_V W_\mu^+ W_\nu^- V^{\mu\nu} + \tag{5}$$

$$\frac{\lambda_V}{m_W^2} V^{\mu\nu} W_\nu^{+\rho} W_{\rho\mu}^- + ig_5^V \epsilon_{\mu\nu\rho\sigma}((\partial^\rho W^{-\mu})W^{+\nu} - W^{-\mu}(\partial^{\rho W^{+\nu}}))V^\sigma +$$

$$ig_4^V W_\mu^- W_\nu^+(\partial^\mu V^\nu + \partial^\nu V^\mu) - \frac{\tilde{\kappa}_V}{2} W_\mu^- W_\nu^+ \epsilon^{\mu\nu\rho\sigma} V_{\rho\sigma} - \frac{\tilde{\lambda}_V}{2m_W^2} W_{\rho\mu}^- W_\nu^{+\mu} \epsilon^{\nu\rho\alpha\beta} V_{\alpha\beta}]$$

with V={Z or A}

In the standard model $g_1^V = \kappa_V = 1$ and all other couplings are zero. New terms will induce deviations from these values. These deviations are listed in the "Correction to the propagators" section.

2.4 Electroweak baryogenesis

Introducing dimension 6 operators can also bring explanations to already existing problems; for example, it can generate corrections to the SM electroweak theory that can reconcile electroweak baryogenesis with the Higgs mass experimental constraints.

Baryogenesis is the general name for "the origin of the matter-antimatter asymmetry in the Universe. Sakharov argued in the 1960's that this asymmetry could happen only if the universe contained the three following elements:

- Baryon number must be violated efficiently, early in the universe
- The discrete symmetries C and CP must be violated
- The universe must fall out of thermal equilibrium, at the precise moment when baryon number switches from being efficiently violated, to being almost exactly conserved.

It turns out that the Standard Model does contain those three ingredient, however it fails to agree with observations :

- The CP violation provided by the phase degree of freedom in the CKM matrix is not large enough to reproduce the observed baryon asymmetry.
- The phase transition in the SM framework is not strong enough ; namely it would require for the Higgs to have a mass around 42GeV, which is ruled out experimentally.

However, adding extra dimension 6 operators can solve this last problem by generating corrections to the Higgs mass, and therefore allowing to relax this constraint :

$$m_H^2 \leq (42GeV)^2 + 8c_\alpha \frac{v^2}{\Lambda^2}$$

with $v = 246$GeV and $\Lambda = 1 - 2$TeV and $c_\alpha \sim O(1)$, we can clearly push the higgs mass to make it agree with current experimental bounds.

3 Computer-based study

One of the goal of my internship was to find out which softwares could be used to extract informations automatically from a given Lagrangian. In other words : is it possible for a software, or a series of software used together, to get the mass spectra, interaction vertices, scattering amplitudes, or even cross sections of various processes from a given Lagrangian ? If this can be achieved for "common Lagrangians", is this still true for more complicated Lagrangians like the non renormalizable ones we are interested in ?

3.1 FeynArts, FormCalc and FeynRules

FeynArts

FeynArts is a mathematica package created by Thomas Hahn that allows the generation of Feynman diagrams and Feynman amplitudes for a given set of interactions and a given topolgy.

With FeynArts, everything starts with a model file; the package comes with a few default model files (standard model, qcd,...) where all the particles and interactions are classified into tables and where all interaction coefficients can be readily modified i.e. it is easy to use and customizable.

After loading this package into mathematica:

$[<<" \ \backslash FeynArts.m"]$

you have to decide the topology of the interaction that you want to study; for instance:

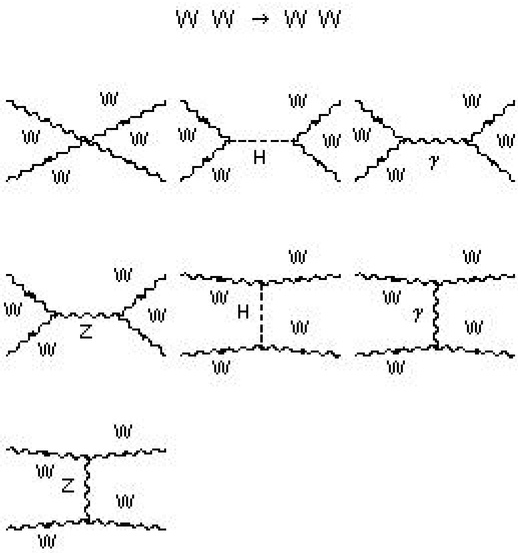
$[T = CreateTopologies[0, 2- > 2]]$

will generate processes with 0 loops, 2 incoming and 2 outgoing particles. It is also possible to exclude internal lines by adding the option $"ExclueTopologies- > Internal"$.

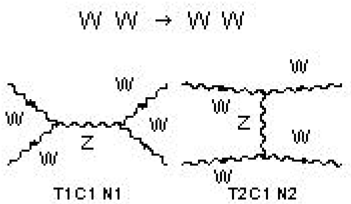
Now this was just to define the topology : to generate the Feynman diagrams for a given process, you have to specify which interactions you want to study; for example:

$[Ins = InsertFields[T, V[3], -V[3]- > V[3], -V[3], InsertionLevel- > Classes]]$

will create the diagram for incoming and outgoing W particles (V[3] in the SM model file):



It is of course possible to pick out specific processes ; adding the option "$LastSelections- > V[2]$" will pick out the processes where a Z-boson is exchanged:



Finally, it is possible to generate the scattering amplitude of this diagram:

$$[Amp = CreateFeynAmp[Ins]]$$

Adding the option "$Truncated- > True$" gets rid of the polarization tensors:

$$\text{FeynAmpList}\left(\text{Process} \to \begin{pmatrix} V(3) & \text{p1} & \text{MW} \\ -V(3) & \text{p2} & \text{MW} \end{pmatrix} \to \begin{pmatrix} V(3) & \text{k1} & \text{MW} \\ -V(3) & \text{k2} & \text{MW} \end{pmatrix}, \text{Model} \to \text{SM}, \text{GenericModel} \to \text{Lorentz}, \text{AmplitudeLevel} \to \{\text{Classes}\},\right.$$
$$\text{ExcludeParticles} \to \{\}, \text{ExcludeFieldPoints} \to \{\}, \text{LastSelections} \to \{V(2)\}\Big)\Big(\text{FeynAmp}\Big(\text{GraphID}(\text{Topology} == 1, \text{Generic} == 1, \text{Classes} == 1, \text{Number} == 1),$$
$$\text{Integral}(), -\frac{1}{\text{SW}^2}\Big(\text{CW}^2\,\text{EL}^2\,((\text{p2}-\text{p1})[\text{Lor5}]g(\text{Lor1}, \text{Lor2}) + (\text{p1}+\text{k1}+\text{k2})[\text{Lor2}]g(\text{Lor1}, \text{Lor5}) + (-(\text{p2})-\text{k1}-\text{k2})[\text{Lor1}]g(\text{Lor2}, \text{Lor5}))$$
$$((\text{k1}-\text{k2})[\text{Lor6}]g(\text{Lor3}, \text{Lor4}) + (-2\,(\text{k1})-\text{k2})[\text{Lor4}]g(\text{Lor3}, \text{Lor6}) + (\text{k1}+2\,(\text{k2}))[\text{Lor3}]g(\text{Lor4}, \text{Lor6}))\,g(\text{Lor5}, \text{Lor6})\,\frac{1}{(\text{k1}+\text{k2})^2-\text{MZ}^2}\Big)\Big),$$

Now, this expression is not really "user-friendly" (it would be much nicer with Mandelstam variables for example) but FeynArts doesn't seem to be

able to generate that.
In otherwords, FeynArts is a nice package to generate Feynman diagrams and get interaction vertices, but the outputs often are poorly written. And a much bigger trouble for us here is that FeynArts is limited to opeartors of dimension D ≤ 4: FeynArts cannot be used to compute non-renormalizable interactions

FormCalc

FormCalc is a Mathematica package for the calculation of tree-level and one-loop Feynman diagrams. It basically just reads diagrams generated by FeynArts and turns the results in a user friendly-way, well suited for further numerical and analytical evaluation. Therefore comibining FeynArts with FormCalc could generate ready-to-use scattering amplitudes.
However, I never succeeded in making it work, despite various attempts. First of all, I tried on my computer (on windows, using X-term). The problem I had was when trying to compile FormCalc 6.0 (or any other version) : the compiler didn't recognize my OS (as it needs to know the OS to create a new directory). I actually managed to bypass that by modifying the code to force it to understand i was working on Windows.

But then another trouble arose : it couldn't find "Readform.exe". I tried downloading it separately, installing "Form" over and over again (even though the installation manual said it wasn't required), but still it wasn't working.

I thought running it on Windows was maybe the problem and therefore I decided to run it on one of the CERN's computer, using "Scientific Linux". The first problem arose again, but i could bypass it to force it to understand I was using Linux (I never figured out why this OS recognition never worked).

But another problem arose : even though mathematica 5.2 and 6 were installed, it said it requires an "mcc compiler" (which usually comes with mathematica) and couldn't find any (I checked and there was no mcc directory or anything that could look like it in the mathematica folder -version 5.2 ans 6-).

It looks lke this problem already happened to some people as on FormCalc's website it is possible to download an already compiled Formcalc/Linux folder if the compiling didn't work. So I downloaded it and ran it in mathematica.

I started mathematica, loaded feynarts package, created an amplitude, then i loaded FormCalc (which seemed to load properly) and when I tried to use the "CalcFeynAmp" command an error occured :

"ReadForm::noopen: Cannot open !/afs/..../Desktop/Formcalc/Form/form-linux /tmp/m34.frm"

I checked and I do have form-linux and m34.frm, and they are at the right locations. So I didn't really understand what was going on..... and after various attempts (which meant rewriting some of the program) I gave up on using FormCalc.

FeynRules

As FeynArts didn't seem to be able to handle non-renormalizable terms, i decided to try another mathematica package called FeynRules. FeynRules was created by C. Duhr and N. Christensen; it can compute the interaction vertices, check the hermiticity, or get the mass spectrum from a Lagrangian, but it does not calculate scattering amplitudes. However, its results can be exported to FeynArts (which calculates amplitudes) or CalcHEP (which calculates cross sections).

Again, you start by loading the package into mathematica and loading the model file you want to use (FeynRules also comes with a few generic and very useful model files). But this time, model files don't contain processes, they contain Lagrangians. For example, one can find in the SM model file the following code :

```
(* SM Lagrangian *)

(******************** Gauge F^2 Lagrangian terms*************************)
(*Sign convention from Lagrangian in between Eq. (A.9) and Eq. (A.10) of Peskin & Schroeder.*)
 LGauge = -1/4 (del[Wi[nu, i1], mu] - del[Wi[mu, i1], nu] + gw Eps[i1, i2, i3] Wi[mu, i2] Wi[nu, i3])*
                 (del[Wi[nu, i1], mu] - del[Wi[mu, i1], nu] + gw Eps[i1, i4, i5] Wi[mu, i4] Wi[nu, i5]) -

    1/4 (del[B[nu], mu] - del[B[mu], nu])^2 -
```

Clearly this model file can be very easily modified and any kind of terms can be added, even non renormalizable ones. It is possible to generate Feynman rules:

$$[vertsGauge = FeynmanRules[LGauge, FlavorExpand\backslash[Rule]SU2W]]$$

which gives all the interaction vertices for all the processes allowed by the Lagrangian in the model file. And it is also possible to get the mass spectrum of a given lagrangian :

$$[GetMassTerms[LGauge + LHiggs]]$$

Therefore, my goal was now to add non renormalizable terms, and evertyhing did work out pretty well.

3.2 Mass corrections

Using FeynRules, we can add $O_s = (H^\dagger \sigma^a H) W^a_{\mu\nu} B^{\mu\nu}$ to the gauge lagrangian, one gets, using the GetMassTerms command line :

```
GetMassTerms[LGauge + LHiggs]
```

$$\frac{\mathrm{ee}^2\,\mathrm{sw}^2\,A_{\mathrm{Index}(\mathrm{Lorentz,mu})}^2\,v^6}{8\,\mathrm{cw}^2\,R^2} + \frac{\mathrm{ee}^2\,Z_{\mathrm{Index}(\mathrm{Lorentz,mu})}^2\,v^6}{8\,R^2} + \frac{\mathrm{ee}^2\,\mathrm{sw}\,A_{\mathrm{Index}(\mathrm{Lorentz,mu})}\,Z_{\mathrm{Index}(\mathrm{Lorentz,mu})}\,v^6}{4\,\mathrm{cw}\,R^2} +$$
$$\frac{\mathrm{ee}^2\,\mathrm{sw}\,Z_{\mathrm{Index}(\mathrm{Lorentz,mu})}^2\,v^4}{4\,\mathrm{cw}\,R} + \frac{\mathrm{cw}\,\mathrm{ee}^2\,Z_{\mathrm{Index}(\mathrm{Lorentz,mu})}^2\,v^4}{4\,R\,\mathrm{sw}} + \frac{\mathrm{ee}^2\,\mathrm{sw}^2\,A_{\mathrm{Index}(\mathrm{Lorentz,mu})}\,Z_{\mathrm{Index}(\mathrm{Lorentz,mu})}\,v^4}{4\,\mathrm{cw}^2\,R} +$$
$$\frac{\mathrm{ee}^2\,A_{\mathrm{Index}(\mathrm{Lorentz,mu})}\,Z_{\mathrm{Index}(\mathrm{Lorentz,mu})}\,v^4}{4\,R} + \frac{1}{4}\,\mathrm{ee}^2\,Z_{\mathrm{Index}(\mathrm{Lorentz,mu})}^2\,v^2 + \frac{\mathrm{ee}^2\,\mathrm{sw}^2\,Z_{\mathrm{Index}(\mathrm{Lorentz,mu})}^2\,v^2}{8\,\mathrm{cw}^2} +$$
$$\frac{\mathrm{cw}^2\,\mathrm{ee}^2\,Z_{\mathrm{Index}(\mathrm{Lorentz,mu})}^2\,v^2}{8\,\mathrm{sw}^2} - \frac{3}{2}\,H^2\,\lambda\,v^2 + \frac{\mathrm{ee}^2\,W_{\mathrm{Index}(\mathrm{Lorentz,mu})}\,W^\dagger{}_{\mathrm{Index}(\mathrm{Lorentz,mu})}\,v^2}{4\,\mathrm{sw}^2} + \frac{H^2\,\mathrm{muH}^2}{2}$$

which, after using the FullSimplify command, gives :

$$M_Z^2 = M_{Z_{SM}}^2 \left(1 + \frac{2cos(\theta_w)sin(\theta_w)}{\Lambda^2} v^2\right)$$

Other mass corrections due to dimension 6 operators are listed below:

Dim6 operator	Mass Correction
$O_s = (H^\dagger \sigma^a H) W^a_{\mu\nu} B^{\mu\nu}$	$M_Z^2 = M_{Z_{SM}}^2 (1 + \frac{2cos(\theta_w)sin(\theta_w)}{\Lambda^2} v^2)$
$O_{WH} = (W^a_{\mu\nu})^2 H^\dagger H$	$M_Z^2 = M_{Z_{SM}}^2 (1 + \frac{2cos(\theta_w)^2 v^2}{\Lambda^2})$ and $M_W^2 = M_{W_{SM}}^2 (1 + \frac{2v^2}{\Lambda^2})$
$O_{BH} = (B_{\mu\nu})^2 H^\dagger H$	$M_Z^2 = M_{Z_{SM}}^2 (1 + \frac{2sin(\theta_w)^2}{\Lambda^2} v^2)$
$O_T = \vert H^\dagger D_\mu H\vert^2$	$M_Z^2 = M_{Z_{SM}}^2 (1 + \frac{v^2}{4\Lambda^2})$
$O_{DH} = H^\dagger H \vert D_\mu H\vert^2$	$M_Z^2 = M_{Z_{SM}}^2 (1 + \frac{v^2}{4\Lambda^2})$ and $M_W^2 = M_{W_{SM}}^2 (1 + \frac{v^2}{4\Lambda^2})$
$O_Y = (\partial_\rho B_{\mu\nu})^2$	None
$O_{3W} = \epsilon^{abc} W_\nu^{\mu a} W_\tau^{\nu b} W_\mu^{\tau c}$	None
$O_Y = (\partial_\rho B_{\mu\nu})^2$	None

3.3 Correction to the propagators : study of the triple gauge boson couplings

3.3.1 WWA vertex

With FeynRules, it is also possible to compute interaction vertices, therefore I also used it to compute the corrections to the WWA vertex induced by dimension 6 operators.

For example, for $O_{BDH} = iB_{\mu\nu} D^\mu H^\dagger D^\nu H$, FeynRules gives you the list of interaction vertices, one of them being :

Vertex 7

Particle 1 : Vector , A

Particle 2 : Vector , W

Particle 3 : Vector , $W^\dagger$

Vertex:

$$-\frac{c_w\, e^2\, v^2\, p_1^{\mu_3}\, \eta_{\mu_1,\mu_2}}{4\, R\, s_w^2} + \frac{c_w\, e^2\, v^2\, p_1^{\mu_2}\, \eta_{\mu_1,\mu_3}}{4\, R\, s_w^2}$$

which can be rewritten : $\frac{g^2 cos(\theta_W) v^2}{4\Lambda^2}(p^\mu \eta^{\nu\rho} - p^\nu \eta^{\mu\rho})$.

For the WWA vertex correction generated by $O_{WDH} = iW^a_{\mu\nu} D^\mu H^\dagger \sigma^a D^\nu H$, FeynRules gives:

Vertex 7

Particle 1 : Vector , A

Particle 2 : Vector , W

Particle 3 : Vector , $W^\dagger$

Vertex:

$$-\frac{e^2\, v^2\, p_1^{\mu_3}\, \eta_{\mu_1,\mu_2}}{4\, R\, s_w} + \frac{e^2\, v^2\, p_1^{\mu_2}\, \eta_{\mu_1,\mu_3}}{4\, R\, s_w}$$

which can be rewritten as follows : $\frac{g^2 sin(\theta_W) v^2}{4\Lambda^2}(p^\mu \eta^{\nu\rho} - p^\nu \eta^{\mu\rho})$

For the WWA vertex correction generated by $O_s = (H^\dagger \sigma^a H) W^a_{\mu\nu} B^{\mu\nu}$, FeynRules gives:

Vertex 13

Particle 1 : Vector , A

Particle 2 : Vector , Wi

Particle 3 : Vector , Wi

Vertex:

$$\frac{c_w\, g_w\, v^2\, \epsilon_{i1,\text{Index[SU2W,Ext[2]]},\text{Index[SU2W,Ext[3]]}}\, p_1^{\mu_3}\, \eta_{\mu_1,\mu_2}}{R} - \frac{c_w\, g_w\, v^2\, \epsilon_{i1,\text{Index[SU2W,Ext[2]]},\text{Index[SU2W,Ext[3]]}}\, p_1^{\mu_2}\, \eta_{\mu_1,\mu_3}}{R}$$

which can be rewritten as follows : $-\frac{g cos(\theta_W) v^2}{\Lambda^2}(p^\mu \eta^{\nu\rho} - p^\nu \eta^{\mu\rho})$

3.3.2 WWZ vertex

As FeynRules lists all the different vertices, it is also possible to get the corrections to the WWZ vertex coming from the dimension 6 operators:

For $O_{BDH} = iB_{\mu\nu}D^{\mu}H^{\dagger}D^{\nu}H$, FeynRules gives

```
Vertex 10

Particle 1 : Vector , Z

Particle 2 : Vector , W

Particle 3 : Vector , W†

Vertex:
```

$$\frac{e^2 v^2 p_1^{\mu_3} \eta_{\mu_1,\mu_2}}{4 R s_w} - \frac{e^2 v^2 p_1^{\mu_2} \eta_{\mu_1,\mu_3}}{4 R s_w}$$

which can be rewritten: $\frac{g^2 sin(\theta_W)}{4\Lambda^2}((p^{\mu}\eta^{\nu\rho} - p^{\nu}\eta^{\mu\rho})$

For $O_{WDH} = iW^a_{\mu\nu}D^{\mu}H^{\dagger}\sigma^a D^{\nu}H$, FeynRules gives:

```
Vertex 10

Particle 1 : Vector , Z

Particle 2 : Vector , W

Particle 3 : Vector , W†

Vertex:
```

$$-\frac{c_w e^2 v^2 p_1^{\mu_3} \eta_{\mu_1,\mu_2}}{4 R s_w^2} + \frac{e^2 v^2 p_2^{\mu_3} \eta_{\mu_1,\mu_2}}{4 c_w R} + \frac{c_w e^2 v^2 p_2^{\mu_3} \eta_{\mu_1,\mu_2}}{4 R s_w^2} + \frac{c_w e^2 v^2 p_1^{\mu_2} \eta_{\mu_1,\mu_3}}{4 R s_w^2} - \frac{e^2 v^2 p_3^{\mu_2} \eta_{\mu_1,\mu_3}}{4 c_w R} - \frac{c_w e^2 v^2 p_3^{\mu_2} \eta_{\mu_1,\mu_3}}{4 R s_w^2} - \frac{e^2 v^2 p_2^{\mu_1} \eta_{\mu_2,\mu_3}}{4 c_w R} - \frac{c_w e^2 v^2 p_2^{\mu_1} \eta_{\mu_2,\mu_3}}{4 R s_w^2} + \frac{e^2 v^2 p_3^{\mu_1} \eta_{\mu_2,\mu_3}}{4 c_w R} + \frac{c_w e^2 v^2 p_3^{\mu_1} \eta_{\mu_2,\mu_3}}{4 R s_w^2}$$

which can be simplified and rewritten as : $-\frac{g^2 v^2}{4cos(\theta_W)\Lambda^2}((k^{\rho}\eta^{\mu\nu} - p^{\rho}\eta^{\mu\nu} - k^{\nu}\eta^{\mu\rho} + cos(\theta_W)^2(q^{\nu}\eta^{\mu\rho} - q^{\mu}\eta^{\nu\rho}))$

For $O_s = (H^{\dagger}\sigma^a H)W^a_{\mu\nu}B^{\mu\nu}$, FeynRules gives $\frac{g sin(\theta_W) v^2}{\Lambda^2}(p^{\mu}\eta^{\nu\rho} - p^{\nu}\eta^{\mu\rho})$.

3.4 LanHEP and CalcHEP

LanHEP is a program developped to convert any Lagrangian you want written in a compact form close to one used in publications in a text file to CalcHEP/CompHEP files. It can also extract all the interaction vertices, and check various aspects of the given Lagrangian.

The output can be either in TeX (to get a list of the interaction vertices, parameters,....) or Comp/CalcHEP format (which we can then use, as we will see, to generate amplitudes and cross sections).

Here is how a common LanHEP file generation looks like:

Step 1: LanHEP input text file

```
model QED/1.
parameter ee=0.31:'elementary electric charge'.
spinor e1/E1:(electron, mass me=0.000511).
vector A/A:(photon).
let F^mu^nu=deriv^nu*A^mu-deriv^mu*A^nu.
lterm -1/4*(F^mu^nu)**2 - 1/2*(deriv^mu*A^mu)**2.
lterm E1*(i*gamma*deriv+me)*e1.
lterm ee*E1*gamma*A*e1.
```

Step 2: LanHEP command line

```
Benjamin Topper@Benjamin ~
$ ./lhep ./mdl/test6.txt -v
```

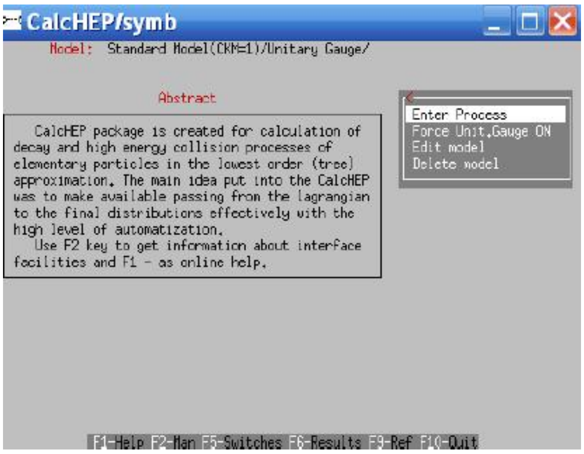
Step 3: LanHEP output (CalcHEP format)

Those files can be directly used in CalcHEP to generate feynman diagrams and cross sections
.

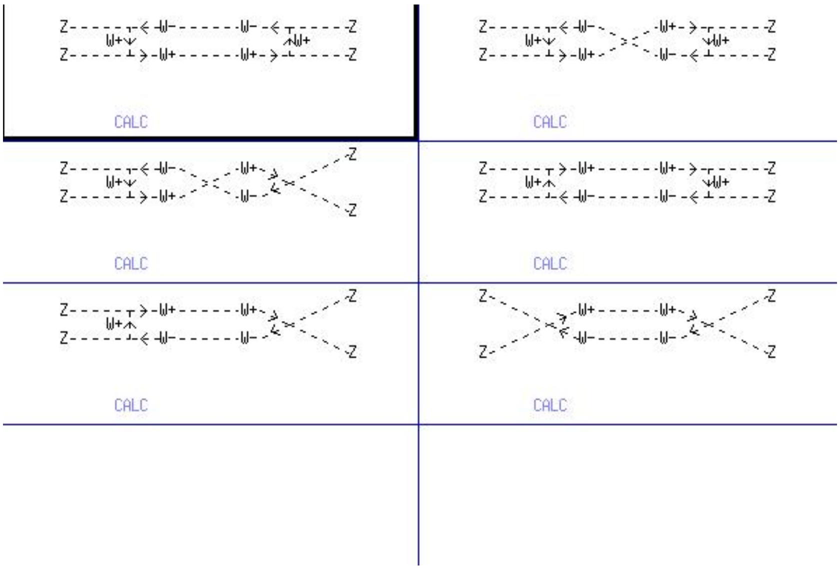
3.4.1 Generating amplitudes with CalcHEP

CalcHEP is a package for automatic calculations of elementary particle decay and collision properties, especially cross sections, in the tree-level approximation. LanHEP allows you to start from any kind of Langrangian (even non renormalizable ones) as an input.

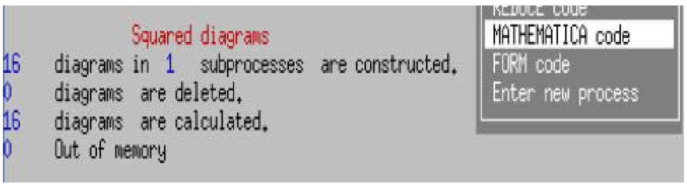
CalcHEP is easy to use because it has a graphic interface :



First you can generate all Feynman diagrams for a given process and select only the one you're interested in computiing :



Then you can write the cross Section in Mathematica format



However, when using the given Mathematica commands C:/cygwin/calchep-2.5/utile/sum-22.m or
C:/cygwin/calchep-2.5/essai/results/symb1.m the output expression is really messy (often a few hundred lines).

Our first goal was to generate Standard Model processes to see if we could extract the scattering amplitudes from these cross sections, so we decided to compute the following processes : WW¿WW, W+Z¿W+Z , ZZ¿W+W-.

The idea was first to find a way to get the square root of the cross sections, but the fact that they were 100lines long or so wasn't making it simple. My supervisor had the idea to simplify the cross section expression only by looking at the first time of the series in the large momentum limit.

```
FullSimplify[Series[sum /. {s → x*s, t → x*t}, {x, Infinity, -2}], CW^2 + SW^2 == 1]
```

This greatly simplified the cross section expression.

However, it turned out that the more recent versions of CalcHEP seem to make a confusion between channels (t-channel should be the u-channel) when switching to Mandelstam variables (actually we are not even sure the problem is as simple as a switch between the different channels; the only obvious thing is that the results given by the new version of CalcHEP are not the same and a switch between t and u could solve that problem).

Therefore I had to use an older version of CompHEP, and I did manage to extract the correct scattering amplitudes for those different processes. For example, for W+Z¿W+Z:

```
FullSimplify[Series[sum /. {s → x*s, t → x*t}, {x, Infinity, -2}]
```

$$\frac{EE^4\,(SW^2-1)^2\,t^2}{144\,CW^8\,MZ^4\,SW^4\,\left(\frac{1}{x}\right)^2}+O\left(\frac{1}{\frac{1}{x}}\right)$$

which gives the correct scattering amplitude for this process after taking its square root.

However, CalcHEP doesn't allow yet to specify a polarization for massive particles (as can be seen with the 1/3 in the scattering amplitude). It automatically averages over initial and summing over final polarizations, and we would only be interested by longitudinal polarizations which are the ones growing with energy.

4 Conclusion

During my internship, I have been looking for different ways to predict the signature of new phyiscs at the order of the TeV scale.

Some of my work was theoretical, in order to understand the physics behind effective lagrangian analysis, and the importance of the various dimension 6 operators we were going to use in our computer study. However, most of my work was to deal with the various computer programs available for High Energy Physics, in order to find the ones that could be used with non-renormalisable lagrangians.
This turned out to be quite tricky, as most computer programs are not designed to work with that kind of lagrangians; most of them can only compute standard model or standard model-like processes and do not allow extra effective operators.

I was able to fully compute the conscequences of dimension 6 operators for masses (i.e how the mass spectrum gets modified), interaction vertices (in this paper, only triple gauge couplings modifications were indicated). I also found a way to compute scattering amplitudes using CalcHEP, which could lead to automatic calculation of scattering amplitudes for non-renormalizable processes.

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5 Appendix : More explicit calculations

In this appendix, i will detail a bit more the different relations that can be derived between the dimension 6 operators that we picked out.

5.1 Multiplication

As already stated, the SM equations of motion are:

$$D^\nu W^a_{\nu\mu} + g(iH^\dagger \frac{\sigma^a}{2} D_\mu H + h.c.) = 0 \tag{6}$$

$$\partial^\nu B_{\nu\mu} + g'(iH^\dagger D_\mu H + h.c.) = 0 \tag{7}$$

However, by multiplying the first equation by $(iH^\dagger \sigma^a D_\mu H + h.c)$ and the second one by $(iH^\dagger D_\mu H + h.c)$, we get the two following equations :
$iH^\dagger \sigma^a D_\mu H D^\nu W^a_{\nu\mu} + iD_\mu H^\dagger \sigma^a H D^\nu W^a_{\nu\mu} + \frac{1}{2}(iH^\dagger \sigma^a D_\mu H + h.c)(iH^\dagger \sigma^a D_\mu H + h.c)$
and $(iH^\dagger D_\mu H + h.c)\partial^\nu B_{\nu\mu} + (iH^\dagger D_\mu H + h.c)(iH^\dagger D_\mu H + h.c)$
It turns out that those two equation can be modified in order to rewrite them in terms of other operators.

In the first equation:
Integrating by parts the first term, we get the following result $g(W^a_{\mu\nu})^2 H^\dagger H + g'(H^\dagger \sigma^a H) W^a_{\mu\nu} B^{\mu\nu} = gO_{WH} + g'O_S$
and the second term gives $4O_{WDH}$.

Using Fierz identities on the third term, it is possible to rewrite it as $6gO_{DH}$.
Therefore we finally get the following relation :

$$6O_{DH} - O_{WH} + O_S - 4O_{WDH} = 0 \tag{8}$$

In the second equation:
The second equation uses the same kind of techniques, it is just simpler as we are manipulating partial derivatives and $(iH^\dagger D_\mu H + h.c)$ instead of $(iH^\dagger \sigma^a D_\mu H + h.c)$. It leads to

$$4g'O_T - gO_S - 4O_{BDH} - g'O_{BH} + 2g'O_{DH} \tag{9}$$

5.2 Bianchi Identities

Using the Bianchi identities:

$$D_\rho W^a_{\mu\nu} + D_\mu W^a_{\nu\rho} + D_\nu W^a_{\rho\mu} = 0 \tag{10}$$

$$\partial_\rho B_{\mu\nu} + \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} = 0 \tag{11}$$

new realtions between operators can be found. For example, manipulating the first equation leads to the following equation:
$(D_\rho W^a_{\mu\nu})^2 + D^\rho W^{\mu\nu a} D_\mu W^a_{\nu\rho} + D^\rho W^{\mu\nu a} D_\nu W^a_{\rho\mu} = 0$
$(D_\rho W^a_{\mu\nu})^2 + 2D^\rho (W^{\mu\nu a} D_\mu W^a_{\nu\rho}) - 2W^{\mu\nu a} D^\rho D_\mu W^a_{\nu\rho} = 0$
And now calculating the commutator between $[D^\rho, D_\mu]$, we finally get:

$$O_W + 3g^2 O_{DH} + 2g O_{3W} = 0 \tag{12}$$

.

We can manipulate the second equation in the same way to get:

$$O_Y - 2g'^2 O_T - g'^2 O_{DH} = 0 \tag{13}$$

5.3 Integration by parts

It can also be shown that integrating by parts some operators can lead to other operators: $O_{\partial H} = (\partial_\mu H^\dagger H)^2 = [(D_\mu H^\dagger)H + H^\dagger D_\mu H]^2$
which therefore gives:

$$O_{\partial H} = -O_{DH} \tag{14}$$

And also:
$O_{HDH} = (H^\dagger D_\mu H)(H^\dagger D^\mu H) = -(H^\dagger D^\mu H) + H^\dagger H |D_\mu H|^2$
which can be rewritten : $O_{HDH} + O_T - O_{DH} = 0$
and using our previous result we get:

$$O_{HDH} + O_T + O_{\partial H} = 0 \tag{15}$$

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